Adaptive IIR Filtering Algorithms for Enhanced CMUT Performance

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Abstract— The use of adaptive filtering as a means of signal processing in sensor applications provides stability and accuracy when operating with sensors that have slowly varying coefficients in their transfer function. This work was conducted using a non-linear state space model of a capacitive micromachined ultrasonic transducer (CMUT) based on FEM data to analyze the simulated effects of adaptive infinite impulse response (IIR) filtering on a through transmission CMUT system. A number of different IIR filter algorithms were investigated and the convergence rates, final mean squared error (MSE) and filter stability among other parameters were analyzed. Included in these algorithms were the full gradient descent method, simplified gradient method, Feintuchs' method, recursive predictor error (RPE) method, orthogonal triangular (QR) decomposition and pseudo linear regression recursive least squares (PLR-RLS). The adaptive IIR filters were applied for system identification, equalization and active noise cancellation (ANC) operations for the study. Exponential convergent approximation time coefficient, a measure of the adaptive filter's ability to track changes, for the ANC case has been shown to vary by more than 20%. MSE variations for the differing algorithms of greater than 10dB have been obtained and filter stability was found to be dependant on a number of internal algorithm parameters, such as the numerator/denominator adaptation ratio, as well as the choice of algorithm.

***Index Terms*— Adaptive IIR, CMUT, system identification, Feintuch LMS**

I. Introduction

CMUTs utilize electrostatic forces to generate a pressure within the operating medium [1]. The direct generation of the pressure wave within the medium allows CMUTs to circumnavigate the requirement for coupling layers as seen in piezoelectric devices. CMUT ultrasonic systems may also suffer from a drift in transfer function coefficients as a result of varying environmental conditions such as temperature [2]. As a consequence the electroacoustic transfer function of these devices can be said to have coefficients that are medium dependant as is evident from an analysis of any of the common analytical models [3]. Although the modification in the device transfer function is deterministic, measurement of all the required input parameters and analytical compensation for drift is impractical.

The problem of variable transfer function coefficients also arises frequently in telecommunications systems, which use a compensation method known as adaptive filtering. This method varies the transfer function of a filter based on an error vector developed from a desired and obtained signal. The methods used are iterative and can thus be viewed as an application of linear prediction. Adaptive filtering in sensor systems allows for system stability in the presence of an uncertain transfer function [4] that otherwise causes the sensor readings obtained to be unreliable. There are two major regimes for adaptive filtering, adaptive FIR and adaptive IIR filtering [5]. Adaptive IIR filters have a number of advantages over their FIR counterparts in acoustic systems as there is an inherent level of recursion [6] and this form of adaptive filter has a smaller footprint for an equivalent order of transfer function to the FIR case [7] which results in miniaturization on integrated circuits.

The use of adaptive filtering in sensor systems has been shown to have a number of advantages [8] with extensive use of FIR adaptive filters, however an analysis of the adaptive algorithms in the IIR case for ultrasonic systems appears to be unpublished. There are a number of commonly used adaptive algorithms that have been shown to be stable, but performance parameters vary depending on the system. The performance of adaptive filters is a trade off between differing parameters including stability, convergence rate and final mean squared error (MSE) [9]. The complexity of the tuning process for the adaptation parameters is also a consideration. With the continued reduction in the cost-function of signal processing power enhanced ultrasonic sensor performance is achievable through the use of digital signal processing in an ever increasing number of applications. This provides the potential for enhanced time of flight readings in systems that are cost sensitive as well as increased robustness with iterative correlation matrix calculation. Adaptive IIR systems may provide performance enhancement across a wide range of ultrasonic applications.

Section II describes the system model used in this work and gives a brief overview of the adaptation functions analyzed and the structural layout of the system. Section III presents the results from the simulation of the various methods and Section IV the conclusions of this work.

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II. Theoretical Background

A. System Modeling

The non-linear of a CMUT used in this work was originally developed by Lohfink and Eccardt [3], which has been shown to be an accurate estimation of the finite element model response. This work was expanded upon by Zhou [10] who developed a state space receiver model. The model is based on a 7.5MHz immersion device with a 300nm air gap and 25.6μm radius membrane. Most 1D CMUT models are based on two differential equations, the first of which is a general expression for the mechanical behavior of a piston in a waveguide:

$$(m_{ps} + m_{fl})\ddot{\omega} + (b_{ps} + b_{fl})\dot{\omega} + k_{ps}\omega = \frac{1}{2}\varepsilon_0 A_{el} U^2 / (h_{gap} - \omega)^2 + F_{ext}, \tag{1}$$

where m is mass, b is damping and k is spring constant, subscript ps is for equivalent piston of the membrane and fl is the fluid interacting with the device, ω is the position of the membrane, U is the voltage across the device, F_{ext} is the incident pressure force on the membrane, h_{gap} is the gap height, ε_0 is the permittivity of free space and A_{el} is the electrode area of the device. The second differential equation describes the electrical behavior of the model and is given by

$$I = \left(\frac{\varepsilon_0 A_{el}}{h_{gap} - \omega} + C_p \right)\dot{U} - \left(\frac{\varepsilon_0 A_{el}}{(h_{gap} - \omega)^2} \right) U, \tag{2}$$

where C_p is the passive capacitance and I is the electric current. The accuracy of these models is sufficient for this work as adaptive filters by their nature do not require precise system modeling for meaningful simulation. The system layout for the three scenarios studied, system identification, equalization and ANC can be seen in Fig 1.

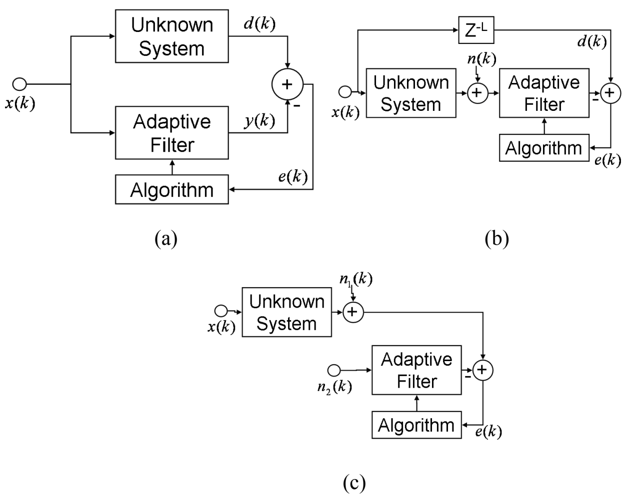

Fig 1: Layout for (a) System Identification, (b) Equalization and (c) ANC

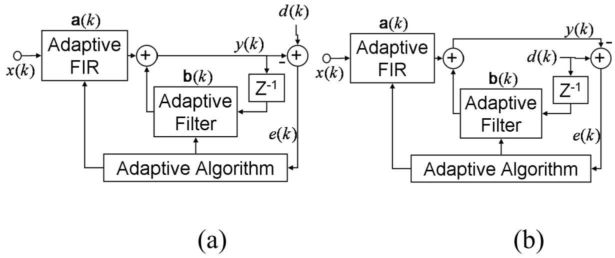

Fig 2: IIR Adaptive Filter Structure (a) Output Error (b) Equation Error

B. Adaptation Functions

To date a handful of adaptive functions have proven to be the most robust and efficient, which include those studied in this work: full gradient descent [11], simplified gradient [11], Feintuchs' method [12], recursive predictor error (RPE) [13], orthogonal triangular (QR) decomposition [14] and pseudo linear regression recursive least squares (PLR-RLS) [15]. All of the methods operate on the filter structure as shown in Fig 2. This structure may be defined by

$$y(k) = \left[\boldsymbol{a}^T(k) \quad \boldsymbol{b}^T(k) \right] \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{y}(k-1) \end{bmatrix} = \boldsymbol{w}_k^T \boldsymbol{z}(k) \tag{3}$$

for the case of output error, where the numerator coefficients ***a(k)*** and denominator coefficients ***b(k)*** at time step k are as shown in Fig 2 and of length N and M respectively. The weight vector w update for full gradient and simplified gradient descent is defined by

$$\boldsymbol{w}_{k+1} = \boldsymbol{w}_k + \mu(-\hat{\nabla}_k) \tag{4}$$

$$\hat{\nabla}_k = \frac{\partial e^2(k)}{\partial \boldsymbol{w}(k)} = -2e(k)\frac{\partial y(k)}{\partial \boldsymbol{w}(k)} = [\alpha_0(k)...\alpha_{N-1}(k), \beta_1(k)...\beta_{M-1}(k)], \tag{5}$$

where μ, the step size, defines the aggression of the adaptation algorithm. For the full gradient descent LMS the $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ updates are defined by

$$\alpha_n(k) = x(k-n) + \sum_{j=1}^{M-1} b_j(k)\alpha_n(k-j), \tag{6}$$

$$\beta_m(k) = x(k-m) + \sum_{j=1}^{M-1} b_j(k)\beta_m(k-j). \tag{7}$$

The simplified gradient descent assumes slowly varying channel coefficients and the update for the $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ vectors are simply given by

$$\alpha_n(k) = \alpha_{n-1}(k-1) \tag{8}$$

$$\beta_m(k) = \beta_{m-1}(k-1), \tag{9}$$

for all values other than α_0 and β_1 which are defined by (6) and (7) respectively. For the Feintuch LMS algorithm the simplifying assumption is made that all derivatives of past outputs with respect to current weights are zero. As a consequence the weight update is given by

$$\boldsymbol{w}_{k+1} = \boldsymbol{w}_k + 2\mu e(k) z(k). \tag{10}$$

The LMS RPE function uses a recursive estimate $\boldsymbol{R}^{-1}(k)$, of the inverse correlation matrix at time step k, with a weight update

$$\boldsymbol{w}_k = \boldsymbol{w}_{k-1} + \mu \boldsymbol{R}^{-1}(k) z(k) e(k). \tag{11}$$

The PLR-RLS weight update is given by

$$\boldsymbol{w}_{k+1} = \boldsymbol{w}_k - \boldsymbol{R}^{-1}(k)\hat{\nabla}_k / 2, \tag{12}$$

the QR RLS algorithm weight update is more involved and given in detail in [14].

III. RESULTS

A. Identification and Impulse Response

When presented with the system identification problem for the system described above and in [3] as is shown in Fig 1, the MSE for the adaptive algorithms as described previously may be seen in Fig 3. It is evident that although the QR RLS algorithm results in the largest single error measurement during its adaptation of all the algorithms the convergence rate is significantly faster and thus convergence is achieved before 0.25x10-5 seconds, significantly sooner than it's nearest and much less stable rival full gradient LMS. The impulse response data for the converged systems however clearly indicates RLS as the best final system estimator with QR RLS being reasonably close in approximation. The impulse response results can be seen in Fig 4 where the system was excited with a Kronecker delta at a simulation time of 5μs, transportation delay is not included in the channel model. All methods show reasonable approximation of the leading peak but only QR-RLS and RLS accurately capture the full pulse shape.

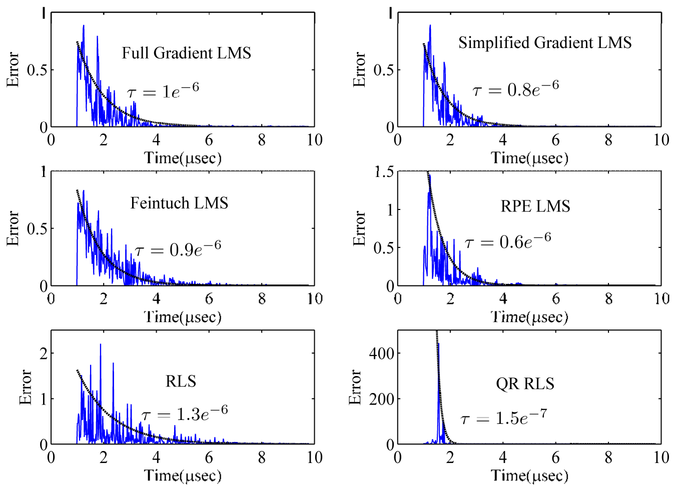

Fig 3: System Identification Error for Various Adaptive Filters

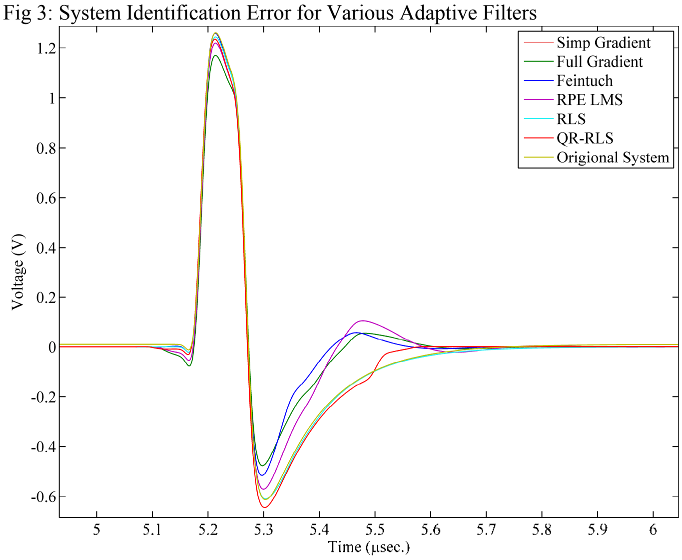

Fig 4: Impulse Response of Identified Systems

B. Equalization

Analysis of the final squared error showed in excess of 10dB difference between the Simplified Gradient and the RLS methods. Using a simple exponential fit to the convergence data in Fig 3 the time constant varies by approx. 31%. In analyzing the MSE for adaptive equalization, as expected, RLS was found to have the lowest final error with QR having the most rapid convergence. The usefulness of adaptive equalization is shown in Fig 5 in which the RLS system has compensated for distortion of a PRBS (Pseudo Random Binary Sequence), this distortion may vary with time. In Fig 5(a) the heavily distorted response characteristic from the input waveform Fig 5(b) is transformed to a close approximation in Fig 5(c). Perfect matching is not achieved for a number of reasons, precision error prevents this with any numerical method but more significantly the non-linear model is used in this work and this will also result in some distortion. Additionally in all adaptive functional approximation there is a compromise between rate of convergence and final settling error. As with the other convergence observations QR outperforms the other algorithms in terms of speed of convergence but the standard RLS achieves a lower final error.

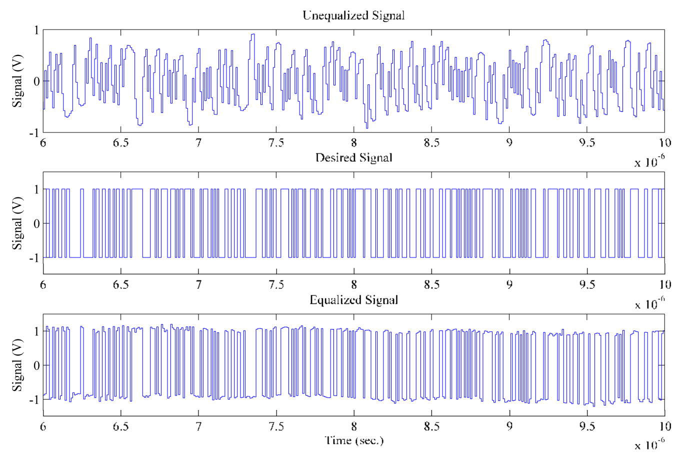

Fig 5: RLS Unequalized, Desired and Equalized Signals

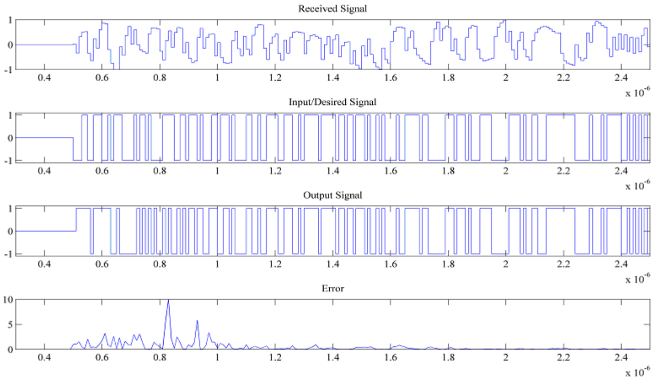

Fig 6: RLS ANC and equalization signals for RLS

C. *Equalization and ANC*

The most important test scenario for the application of adaptive IIR methods on the modeled system is with both equalization and ANC. In this scenario a noisy channel (noise level of –8dB) with channel effects is compensated for by the adaptive system either as a means of monitoring varying channel coefficients or noise. The results of the adaptation of the RLS case, which was found to be the most easily tuned and most stable design once again, are shown in Fig 6. Using a zero-crossing detector results in perfect signal reconstruction for the simulated conditions using the RLS algorithms after 10 bytes of data are transmitted across the channel as shown in Fig 6.

The convergence of the numerator and denominator coefficients for the RLS algorithm, which was found to be the optimum for this application, can be seen in Fig 7. It is clearly observable that following coefficient convergence, drift around the optimum is found. More sophisticated adaptation algorithms can overcome this with variable μ adaptation, but for the purposes of this work this oscillation is not a concern. It was found that filter order, as expected, increases the accuracy of the approximation but only up to a point, after which the affect is negative. This turning point is dependant on the error surface and thus is not easily quantified.

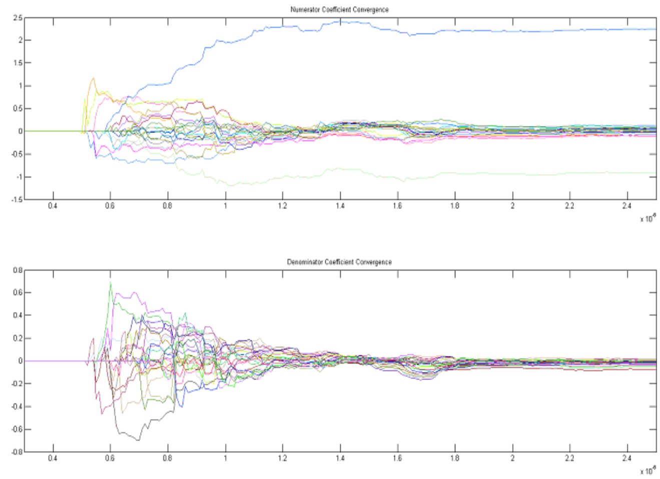

Fig 7: RLS coefficient convergence for ANC and equal.

IV. Conclusion

The convergent parameters have shown the advantages of each of the differing algorithms depending on the designer's requirements. The QR algorithm was found to outperform the other algorithms on most metrics, most notably the convergence rate but not all, with the RLS algorithm outperforming it on final error. There are a number of situations that the designer should be aware of where the more easily implemented algorithms are in fact the better choice, e.g. whether the implementation is to be in software or hardware, and many of the algorithms were designed around one implementation or the other.

Acknowledgment

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